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Two papers on

APPLICATIONS AND MODELLING IN THE MATHEMATICS CURRICULUM

by Mogens Niss

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Two papers on APPLICATIONS AND MODELLING
IN THE MATHEMATICS CURRICULUM

by Mogens Niss

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ABSTRACT:

This text consists of two papers on applications and modelling in the mathematics curriculum. Both papers were presented as plenary lectures at conferences held in the autumn 1991.

The first paper, **Assessment of Mathematical Applications and Modelling in Mathematics Teaching**, was delivered at ICTMA 5 (the Fifth International Conference on the Teaching of Mathematical Applications and Modelling), Noordwijkerhout, the Netherlands, 9-13 September 1991.

The second paper, **Applications and Modelling in School Mathematics - Directions for Future Development**, was delivered at the Third UCSMP (University of Chicago School Mathematics Project) International Conference on Mathematics Education, University of Chicago, 30 October - 1 November 1991.

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Plenary lecture delivered at ICTMA 5, 9-13 September 1991, Noordwijkerhout, the Netherlands.

To appear in the proceedings of the conference.

ASSESSMENT OF MATHEMATICAL APPLICATIONS AND MODELLING IN MATHEMATICS TEACHING

by **Mogens Niss**, IMUFA, Roskilde University, Denmark

Summary
For application and modelling activities to be taken seriously in curricula making use of formalised assessment, it is necessary to have these activities subject to some form of assessment. The leading question for this paper is "How to shape and practice assessment in the area of applications and modelling in such a way that assessment serves its purposes without destroying the application and modelling work?".

The paper addresses this question by dealing with the following further questions: "**What** should be assessed?"; "For **which kinds of tasks and activities** should assessment be conducted?"; "How should assessment be **designed and organised**, and which **assessment modes** should be applied?".

It is concluded that appropriate assessment modes doing justice to the nature of application and modelling work **do exist**, but for their implementation it is necessary that certain traditional requirements to assessment in mathematics education be abandoned.

1. Introduction

Let us begin by stating a few **working definitions.**

(a) We shall adopt the **distinction** between 'assessment' and 'evaluation' suggested at the ICMI Study Conference on "Assessment in Mathematics Education and Its Effects": 'evaluation' concerns the judging of systems, programmes and institutions, whereas 'assessment' concerns the judging of student performance, whether made individually or in groups. Following this distinction, in the present lecture we shall deal with assessment only.

(b) The term 'assessment' will be taken as a **general term**: it encompasses all specific concepts, categories and modes of assessment, including formal as well as informal, continuous as well as discrete, formative as well as summative, classroom assessment and final examinations, tests etc.

(c) We shall refrain from going into details to define mathematical applications and modelling (applications and modelling, for short) in relation to mathematics teaching. Whenever mathematical concepts, methods and results are used to describe, understand or handle aspects of the world outside mathematics, we are to do with an application of mathematics. Any application of mathematics presupposes or implies the construction (be it explicit or implicit) of an image, consisting of mathematical objects, of that segment of the extra-mathematical world to which mathematics is being applied. We shall call such an image 'a mathematical model'. The process of establishing a model will be called 'mathematical modelling'. What interests us in the present context is the inclusion of aspects of applications and modelling in mathematics teaching.

A final remark serves to clarify the character of the present paper:

(d) In most cases it is possible to approach a theme in mathematics education in at least two different ways, a **descriptive/analytic** and a **normative** way. Basically, and formulated in simplified terms, a descriptive/analytic discourse deals with what **is** and how it can be understood, whereas the fundamental interest of a normative discourse is what **should be**. There is no dichotomy between the two kinds of discourse as long as conceptually they are kept clearly apart. For instance, the gap between 'what is' and 'what should be' may be bridged by giving analytic answers to the question 'what **could be**?'. The main interest in this paper concerns **normative issues** but it is endeavoureded to deal with them mainly by **analytic means**.

2. Why is the theme of this lecture interesting?

Why is the relation between 'assessment', on the one hand, and 'applications and modelling', on the other hand, of relevance and interest to us?

> **(1)** If one's interest is **focussed on assessment** in general applications and modelling simply constitute a particular domain - whether new or old - for the exercise of that interest and for activities connected to it.
>
> We could say that with this focus what matters is **the significance of applications and modelling for assessment**.
>
> **(2)** If one's interest is **focussed on applications and modelling** the situation is different. In that case assessment needs only to be considered to the extent it is relevant for applications and modelling, and their roles and lives in the curriculum.
>
> Dually to (1), this focus makes **the significance of assessment for applications and modelling** the main point.

In the present context we shall adopt **focus (2)**. Of course, this does not make focus (1) irrelevant in other contexts. In what follows we shall attempt to maintain a clear distinction between focus (1) and focus (2), though this might turn out to be more difficult in practice than in theory.

If we proceed on the basis of focus (2) **why**, then, **is assessment of relevance and interest to applications and modelling**? For the following reasons (among others, perhaps):

> * - Whether or not a mathematics curriculum that includes application and modelling work is subject to formalised assessment (of whatever type), explicit or implicit notions and criteria of quality are needed for the planning, carrying out, and regulation of the application and modelling work.

Irrespective of all well justified reservations one might have towards existing assessment philosophies and practices, and their consequences, everbody will gain from having **notions and criteria of quality articulated, discussed and analysed**. Even if this does not happen quality will not cease to be judged. Instead, judgement will be exercised implicitly rather than explicitly, and perhaps with intellectual unclarity rather than clarity. (This, of course, is a more general consideration the validity of which is not limited to the area of applications and modelling.)

* - In an educational system that makes use of formalised assessment (and the vast majority of educational systems do), there is a tendency that only **components which are subject to assessment are taken seriously** by students, and by the teachers too, for that matter.

Stated in catch-words: **'what you assess is what you see'**.

Thus, if application and modelling work is to be taken seriously in and by such an educational system it has to be subject to assessment in some form or another.

* - If, in a system making use of formalised assessment, there are principal or practical difficulties in making application and modelling work subject to assessment, an unintended barrier to the acceptance and inclusion of applications and modelling in the curriculum is created. So, answers to the assessment challenge which are educationally satisfactory and practically implementable may be important for the **breaking down of unintended barriers** to including applications and modelling in the mathematics curriculum.

* - If we combine the above observation 'what you assess is what you see' with the classical slogan 'what you see is what you get' we arrive at a new catch-word statement: **'what you assess is what you get'**:

The actual shape and administration of assessment modes and practices significantly influence the conditions for application and modelling work, for better and for worse. The actual shape and administration therefore deserves attention and control.

3. What are the main issues and challenges concerning assessment of applications and modelling?

It is a general problem - and an old observation - that the mere presence of assessment, irrespective of the modes and practices through which it materialises itself, influences the activity that is assessed. The more extensive and complex this activity is, the more delicate balances does it contain, and the more influence does assessment exert on it, also in non-desirable ways. Application and modelling work certainly is of this nature, a nature that makes it particularly vulnerable to most of the forms of assessment in current use. As a matter of fact, these forms of assessment even seem to make it difficult for application and modelling activities to thrive in the curriculum.

If we add that the entire field of assessment implies a host of intricate and intellectually controversial issues and a lot of conflicting interests of those involved (students, teachers, school authorities, society), we are facing important challenges. Pointedly put we can ask the following basic question:

How to shape and practise assessment in the area of applications and modelling in such a way that assessment serves it purposes without destroying the application and modelling work?

If we take this point of departure, as we suggest, our task is not to arrange applications and modelling activity in accordance with - and limited by - existing assessment practices. On the contrary, such activities should be selected, designed and organised to pursue educationally valuable and important goals, and paying respect to what is possible in practice. And **then** assessment should be organised accordingly. To put it briefly: applications and modelling as a component of mathematics education should be granted **priority** over assessment. The dog should wag its tail, not the other way round.

The basic but general question posed above may be coined as follows: how to create modes of assessment which can manage the following tasks:

to

- help the teacher in her/his **planning and carrying out of teaching activities in applications and modelling** in such a way that the students gain something from them that is educationally important ;

- help to **inform**, in a constructive way, **students** (and their teachers) about their strong and week points thus assisting them in gaining new land;

- **create acceptable compromises** between ideal educational considerations orientated towards the individual student, on the one hand, and, on the other hand, society's interest in (a) making decisions, based on assessment results, of wide implication to the future life opportunities of the individual student and to his perception of himself, and in (b) controlling - through the assessment system - curricula, teachers, and teaching institutions.

- **respect** and **not distort**, perhaps even to support, the **entirety** of working processes, forms of organisation, content components, and student attitudes which are characteristic to application and modelling work.

To be more **specific and concrete**, we have to answer the following questions concerning assessment of application and modelling work:

* **Why**, i.e. for which specific purposes, should we assess applications and modelling?

** **What** should be assessed in the applications and modelling sub-space 'content x product x process x...'?

** For which **kinds of task and activity** in applications and modelling should assessment be conducted?

* **Who** should be assessed (individual students, groups, classes)?

* **When** should assessment take place (continuously, discretely, at the end of course/school)?

** **How** should assessment be designed and organised for various purposes, i.e. **which modes of assessment** should be used, **which outcomes should be recorded**, and **how should they be reported?**

* **By whom** should assessment of various kinds be designed, organised and carried out, respectively?

Most of these questions have a general scope. They are not specific to the area of applications and modelling but are nonetheless highly relevant to it.

4. What answers can we give to the challenges?

Of course, it would be futile to attempt in one paper to provide answers to all the questions stated in section 3. In the sequel we shall confine ourselves to dealing with only a few of these questions - those marked with two stars - which appear to be particularly intriguing. We have no pretension of giving a systematic treatment of the issues we shall be addressing nor of being exhaustive in the answers suggested.

A.

** **What** should be assessed in the space 'content x product x process x...' as far as applications and modelling are concerned?

Firstly, let us remind ourselves that the area of applications and modelling is **not a subset of mathematics**. Any instance of application and modelling necessarily contain aspects belonging to the world outside mathematics.

It is not uncommon to encounter the point of view that it may be all right to deal with applications and modelling in mathematics teaching provided that the emphasis is on the mathematical aspects rather than on the extra-mathematical ones. According to this view, the latter should not be made an object of substantial work in the mathematics classroom, perhaps because the mathematics teacher may not be competent to deal with them, or because it would take costly time away from proper mathematical activities. Instead, the extra-mathematical components have to be taken for granted or be taken care of by other subjects.

Although this discussion is not in focus of the present paper, we have to say that in our opinion the point of view just sketched is wrong, and we suppose that there is no need to argue much about it here. The area of applications should not be included in the mathematics curriculum just to create a new platform for doing pure mathematics (not that there is anything wrong with pure mathematics, on the contrary). It should be included because it is educationally valuable in its own right. And for its educational value to blossom its specific character needs to be taken seriously. For applications and modelling to be taken seriously in mathematics teaching, all essential components and processes of application and modelling are to be taken seriously as well.

The validation of models is an essential component in the area of applications of modelling. Since applications and modelling do not belong entirely to the realm of mathematics, the quality of a piece of modelling cannot be judged on mathematical grounds alone. Naturally, the quality of the mathematics involved is crucial, but 'quality' should not be confused with 'level of sophistication'. Excellent models may rely on simple mathematics, and very bad models on highly sophisticated mathematics.

What matters is the extent to which the resulting model is capable of adequately representing that segment of the extra-mathematical world which it is built to represent, with due respect being paid to the purpose for which it is built. Thus, the criteria for good mathematical modelling are part of the criteria for good science.

In discussions on applications and modelling one often encounters the standpoint that validation of a model takes place by confronting it with a given set of data: If the results generated by the model are in reasonable accordance with that set of data (where 'reasonable acccordance' needs to be specified in some way or another) the model is accepted, otherwise it is rejected. This notion of validation is entirely insufficient and unsatisfactory, philosophically as well as scientifically speaking. Philosophically, because it makes 'reality' identical to data in a naïve way, and in a totally misleading way if a **given** set of data is considered the ultimate reality. This notion misses the deep problems involved in the design of experiments, not only involving the collecting of data, and the subsequent validation and interpretation of them - nothing of which is a simple matter - but also involving a complicated dialectical interplay between model, theory and experiment. Scientifically, because it is often very easy to modify or calibrate a model - for instance by adjusting parameters in it - in such a way that it reproduces the given data with mathematical exactness.

Now, what has all this to do with **assessment** as related to the **teaching** of applications and modelling? A whole lot. Repeated application of the 'taking seriously' argument (components not assessed are not taken seriously) leads us to conclude that in mathematics teaching **we ought to assess the entire process of application and modelling in all its phases**, not just the specifically mathematical parts of it. In so doing, we should be prepared to particularly **validate the 'scientific' quality of models constructed by students** and, furthermore,to **assess students' validation of models and applications**, whether constructed by themselves or 'only' critically analysed by them.

What has been said so far in this section does not exhaust, of course, **what** should be assessed in the space 'content x product x process x ...'. We should also assess a lot of other domains in that space. As they are probably fairly obvious, let us just indicate a few of them by listing questions that ought to be asked in the assessment process:

- how far did the students get, relatively to their point of departure?

- in what way and to what extent was teacher or literature assistance used in the process?

- is the correspondence between the issues raised at the beginning of the application and modelling activtity and the answers given at the end reasonable, and are the answers given well justified?

- how is the outcome of the work recorded and communicated? Is it possible to follow the investigation undertaken and the arguments provided from the perspective of analysis and critique ?

- to what extent have the students acquired insights and skills that are transferable to other kinds of situations?

B.

** For what **types of tasks and activities** should assessment of applications and modelling take place?

Naturally, it does not make much sense to assess application and modelling work through tasks and activities which do not pay full respect to the nature of application and modelling work and allow for an appropriate unfolding of a representative set of the content and process components which are characteristic to it. So, the general answer to our question is in a way very simple, yet still general: Assessment should take place for **tasks and activities which do justice to application and modelling work.** This ought to be a trivial answer, but in view of the assessment practices actually applied in many places in relation to applications and modelling it is not.

Many types of tasks and actitivities do not do justice to the nature and characteristics of application and modelling work, because they do not make room for either essential **content** components or essential **process** components of applications and modelling.

Regardless of their intended content, most **short term** tasks and activities (lasting, say, no more than a one digit number of hours) cannot do justice to application and modelling work, simply because of their duration. As has been pointed out by many mathematics educators with extensive application and modelling experiences, this rules out most tasks and activities which are to be exercised under unmodified test and exam conditions with usual time constraints. To the extent it is necessary to subject application and modelling work to assessment under such conditions (and in many countries this is the case), some amount of ingenuity is needed to find ways to remedy the situation. We shall return to this matter in the next session.

As a matter of fact we do have a variety of tasks and activties suitable for applications and modelling and hence for assessment. Most of the operate on various kinds of coursework. To name a just a few: extended investigations, student portfolios, and projects. One particularly appropriate type is **projects.** In the remainder of this section we shall concentrate on projects, in particular on a special type: problem-orientated projects.

Without spending too much time on defining and describing problem-orientated projects let us briefly outline their main features.

Problem-orientated project work is performed by students, individually or in groups, with the aim of investigating a scientific or a practical problem area. The problem area may range from a one-member-set, containing one specific, concrete and well-defined problem, to a very complex set of interrelated problems of a high level of abstraction. The duration of a project is longer than short. It may vary from a few days to several months. Dependent on purpose and content of the project, its product may take different forms. The product may be a written report of some kind: a textbook or a popular book, an article for a scientific journal, a newspaper or a magazine. It may also, less frequently, perhaps, be (the design of) an exhibition, a film, a lecture, a photo slide show, a radio or tv-programme etc.

As far as applications and modelling are concerned, two 'pure' types of projects are of particular relevance:

(a) Projects in which students **examine** applications and modelling work already done by others.

(**b**) Projects in which students **construct** a model themselves.

In practice, projects composed as **mixtures** between the two prevail. The product of such a project will normally be a written report.

We do not know of any kind of task and activity that is better suited to let genuine application and modelling work flourish than problem-orientated projects, whether of type (a), (b), or a convex combination of the two.

As a consequence of the general analysis presented above, projects constitute a reasonable category of task and activity to be subjected to assessment. It is true that assessment of projects has to be based on modes and practices which differ from the conventional ones, but this does not at all make assessment of projects impossible. In fact, in a very flexible way assessment of projects allows us to emphasise varying aspects of applications and modelling in the space 'content x product x process x...'.

C.
** **How** should assessment be designed and organised for various purposes, i.e. **which modes of assessment** should be used, and **which outcomes should be recorded**, and **how should they be reported?**

We are prepared to risk our skin by claiming that assessment of applications and modelling is **easy**. As mentioned earlier, assessment is not easy if we (have to) stick to conventional modes and practices. In that case sound assessment is rather very difficult if not impossible. Assessment of applications and modelling is only easy if we are courageous enough to abandon certain prevalent requirements, such as: assessment should take place under test conditions with time constraints; assessment should be inexpensive, i.e. assessors should not spend more than a few minutes per student; assessment should be a large scale industry (whether commercial or not) making use of standardised schemes which imply a uniform treatment of students; assessment modes should not leave room for different assessors to disagree on the performance of a given group of students or on a specific product; assessment modes should allow for easy marking and grading of student performance. Many of these requirements are instances of one general requirement: assessment should be "objective", i.e. **reliable**. To obtain this it is often considered an acceptable cost that assessment may not be **valid**.

The fear of subjectivity and insistence on objectivity in assessment seems to be more widespread among mathematics educators and mathematicians than is seen with professionals from other subjects. So, mathematics educators and mathematicians are often led to operate with very explicite and rigid categories and criteria, also in cases where this is not reasonable or appropriate. However, this general attitude does not prevent them from having clear ideas about how to judge research papers, books etc. without relying on any formal framework. Moreover, they are very good at being in disagreement with each other on these matters.

We should accept that assessment of applications and modelling has to be exercised as an intricately balanced judging of a vast variety of components in a complex and often fuzzy structure. This implies elements of subjectivity and disagreement, and it implies that assessment takes time and cannot be standardised. It does **not** imply that assessment cannot be exercised on a sound foundation of reflection and reasoning and articulate criteria and be subject to clear communication. It also does not imply that assessment cannot be summative and, if necessary, result in marks that may be given in ways which are fairly robust to change of assessors.

Assessment of applications and modelling should simply be viewed as parallel to assessment of other complex structures in the world. Teachers of mother tongue composition have lived with that complexity for decades, as have assessors of research papers. Why should we in mathematics education not be able to live with it as well? This is a necessary prerequisite for convincing authorities, politicians, employers, parents, students, teachers etc. that things have to be changed in the area of assessment if we want essential aspects of mathematical activity to be made subject of assessment in a reasonable way. We should fight for a much wider recognition of assessment to be based on coursework. It is interesting to notice that in most cases where applications and modelling have been granted a significant position in the mathematics curriculum, assessment is based on coursework. To the extent conventional centrally administered assessment practices are involved it is seen as a necessity, sometimes even as a necessary evil, rather as an asset.

To exemplify what we have in mind, let us describe how we **assess projects** at Roskilde University in Denmark. At that university projects are made by groups of up to 10 students who spend half of their study time in 3-4 months on making a project which is finished by a written report of 50-150 pages. The project report is submitted to an external examiner - and to the project advisor too, of course - who are given two weeks to read it carefully and with critical eyes.

After two weeks the group of students present themselves to an **oral examination** based on the project report and directed by the project advisor and the external examiner. Immediately prior to the oral examination the project advisor and the external examiner meet to discuss the strong and weak points of the project **report**, identify possible issues of particular interest that should be raised at the oral examination, and to agree on a summative level, indicated by an interval of marks (the scale of marks in Denmark contains 10 grades). In very rare cases agreement cannot be reached at this stage, and the decision on marks is postponed till after the oral examination.
Basically, the oral examination is seen as a complement to the assessment of the written report which is considered to be the most important part of what the students have accomplished. The objective of the oral examination is: (1) to examine whether the group as a whole as well as the individual member of it can defend their work well enough to convince both examiners that the students have actually made the project themselves and possess genuine insight into what they have written, and that each student has made a substantial contribution; (2) to investigate the breadth and depth of the insight, knowledge and skills acquired by the individual student in relation to the theme(s) and problem(s) of the project. Both objectives are pursued through a brief presentation made by the students followed by a rather intensive cross-examination of the group and its members. The normal duration of the oral examination is 30-45 minutes per student depending on the number of members in the group.

When the oral examination is over the examiners reconsider the project which now consists of the written report plus the outcomes of the oral examination. Now they combine, in a non-formalised way, their judgment of the written report with their judgment of the quality of the presentations and responses given by the students during the oral. On that basis they agree on the marks to be given to the individual student. The voting procedure takes, say, half an hour. When the examiners have reached a decision on the marks the students are informed about the result and reasons for the decision are presented to them.

In normal cases, in which all the students have been able to demonstrate that they are "the rightful owners" of the work they have presented, the resulting marks do not deviate strongly from the initial mark level stipulated for the written report. The normal range of variation is plus/minus 1-2 marks. The variation between the individual students' marks in such a case is of about the same size. It may happen that the oral defence is either much weaker or much stronger than the written report. If so, assessment leads to marks that deviate accordingly from the one originally given. It may also happen that the oral examination reveals that some members of a group are not rightful owners of a project which is otherwise all right. In that case these students are given non-pass marks whereas their fellow students obtain pass marks.

There is no doubt that this assessment mode is somewhat time consuming, and that it implies elements of subjectivity. It may also be said that it relies on local tradition and tacit knowledge etc. But nobody should claim that it is not thorough and serious. The most important thing, however, is that it is valid because it really does pay due respect to the object that is assessed.

What has just been described is hardly unique. It is, however, far from being an example of mainstream assessment. There are no principal barriers which prevent from becoming mainstream, but ideological, practical and financial barriers abound. In most countries **final written examinations papers** sat by the individual student in spiritual isolation for a restricted number of hours simply is a fact of life. We cannot define that fact away over night. Given those boundary conditions, how can we devise assessment modes which apply to application and modelling work in a reasonable way?

Let us look briefly at a couple of examples of how people in different places have responded to this challenge.

In the Netherlands Jan de Lange and his collaborators at the Freudenthal Institute (formerly OW&OC) have developed what they call the two-stage-task. In the first stage students sit a time restricted written examination paper containing open questions and essay questions for instance on applications and modelling items. The teacher scores the student's papers and hands it back to him/her with information of the scores and of the biggest mistakes. In the second stage the student is given ample time, say several weeks, to rework the task in any way he or she wants. The reworked task is scored by the teacher, and the student is provided with a pair of marks, one for the first stage and one for the second stage.

As part of a curriculum project in Northern Ireland conducted by Ken Houston and others, students are given a collection of materials some weeks before they sit a written examination paper which is based on this material. The idea is to familiarise students with the universe dealt with in the written papers. Something similar may be found elsewhere (for instance in Australia and Portugal), sometimes with the difference that it is the teacher who familiarises students with certain aspects of the theme within which written papers are formulated.

It is characteristic to these and most other responses that they try to circumvent the time restriction and the spritiual isolation of the written examination situation. It does so by combining the written examination with tasks carried out under much less stressing circumstances. In our view this is fine and we should not stop to invest ingenuity in developing new ideas in the same direction. However, we should also not forget that we are dealing with artificial constructs rather than with the "real thing".

5. Concluding remarks

Much of what has been said above about assessment of applications and modelling may be said about the bulk of high level and complex mathematical activity. The general situation is not satisfactory but something can be done to improve it:

* We should involve ourselves in attempts to convince people from different quarters that most current assessment modes are inappropriate to assess high level and complex mathematical activities including applications and modelling. Appropriate modes of assessment for such activities do exist, but they are incompatible with certain traditional requirements to assessment in mathematics. Therefore those requirements have to be abondoned as far as these activities are concerned.

* While waiting for this to happen we need to continue to invest ingenuity in remedying the situation by devising and developing new clever ways to circumvent the trammels created by current assessment modes and practices.

Mogens Niss, final version 27 December 1991

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APPLICATIONS AND MODELLING IN SCHOOL MATHEMATICS - DIRECTIONS FOR FUTURE DEVELOPMENT

by **Mogens Niss**, IMFUFA, Roskilde University, Denmark

1. Introduction

The starting point for the present paper is an assumption supported by empirical evidence: applications and modelling in school mathematics are here to stay.

Less than two decades ago, the inclusion of application and modelling activities in post-elementary school mathematics was still a case that needed a strong advocacy to be taken seriously by the mathematics education community at large. Such an advocacy was provided by mathematics educators and educationalists who saw themselves as forming a vanguard for re-considering and re-shaping the role and character of school mathematics in order to make it capable of offering a relevant mathematical education to the general population in every country.

Today, the situation is changed. If defined in broad and liberal terms, applications and modelling have gained some foothold in mathematics curricula at all levels in most parts of the world since the mid-seventies. This does not imply that there is universal agreement on the definition and practical interpretation of application and modelling activities, nor that their formal position and actual implementation in mathematics curricula are everywhere the same, far from it. What it does imply is that the mathematics education community, curriculum planners and authorities, textbook authors etc. recognise the need for curricula to deal with aspects of applications and modelling in some form or another. It also implies that, in contrast to what was the case twenty years ago, a large variety of materials and resources (e.g. application orientated textbooks, case collections, and computer software) are now available for any educational level.

In view of the position now being held by applications and modelling in school mathematics, I think that instead of asking questions such as: "what can we do to ensure applications and modelling a fair place in mathematics curricula?", time has now come to consider a somewhat different question: *"what new directions will or should future development of applications and modelling in school mathematics teaching take?"*. This, then, will be the main issue to be addressed in the present paper. Although the emphasis will be on secondary school mathematics much of what is going to be said will hold equally well for primary and tertiary mathematics.

In order to create a basis for dealing with our issue, we shall begin our analysis by surveying briefly where we *are* as far as applications and modelling in school mathematics are concerned.

2. The present state of applications and modelling in school mathematics

It was stated above that applications and modelling have now gained a foothold, of some kind or another, in mathematics curricula in most parts of the world. What this means in relation to school mathematics is simply that time and again extra-mathematical situations are subject to mathematical treatment in the mathematics courses. There is, however, considerable variation in how this is done, as regards content, extent, approach, and student activity. For each of these four dimensions, we shall briefly outline the variation one may encounter.

First of all the **interpretation of the term** "an extra-mathematical situation" is very wide indeed as far as **content** is concerned. The only common requirement in all interpretations is that some issues, problems, concepts, objects, phenomenae etc. *not belonging to the world of mathematics* have to be involved in the situation. So, extra-mathematical situations range within a continuous spectrum. At the one end we find tasks which are, in fact, purely mathematical but *dressed up* in a non-mathematical language, at the other end situations which are *authentic* to a subject or practice area outside mathematics by containing genuine problems or objects actually belonging to that area. In between we find all degrees of situations which are cut-out, simplified and idealised versions of ones which are or could be real to other subjects or practice areas.

As to the **extent** to which a given mathematics curriculum includes applications and modelling, there are curricula where extra-mathematical situations are dealt with only occasionally, as islands in an otherwise "purely" mathematical course, either to motivate a new mathematical concept or topic, or to show the relevance to the world outside of some segment of mathematics. But there are also cases where the treating of extra-mathematical situations occupy a much more prominent position, for instance due to a frequent dialectical interplay between work on such situations, and sequences of mathematical theory development. It may also happen that substantial parts of a mathematics course are devoted to the treating of larger applicational situations.

Several different **approaches** to the inclusion of extra-mathematical situations in mathematics courses may be found. In some cases the extra-mathematical aspects of a situation are taken for granted. These aspects are not subject to further examination or activity, and all relevant information about them is assumed to be present already. In other cases it is part of the job to critically examine given information on the extra-mathematical aspects or to procure (additional) information to assist the mathematical treatment.

Sometimes the extra-mathematical situation is *closed* and the problems to be dealt with are posed from the outset. Sometimes the situation is *open-ended* and it is part of the job to select, pose and specify the problems to be tackled. In some cases a *well-defined* collection of mathematical tools are to be activated, in others *any* available tool which is considered relevant and effective in relation to the extra-mathematical situation may be applied.

As finally regards the **spectrum of student activity** utilised to working with extra-mathematical situation, activities range from *passive acquisition* by the individual student of knowledge provided by written expositions or oral presentations directed by the teacher, to *active and independent student work*, perhaps in groups, on projects based on material procured or created by the students themselves.

The *total variation*, constituted by the variation in each of the four dimensions as just sketched, actually encountered between school mathematics curricula having applications and modelling on their agenda is thus tremendous. Only rarely do we find curricula which lie at "the ambitious end" in all four dimensions, but such cases do exist. The typical curriculum which has included some aspects of applications and modelling rather lies at *"the cautious end"*:

It is based on a liberal interpretation of an extra-mathematical situation and operates mostly on cut-out, simplified and idealised versions of (potentially) realistic situations which are presented succinctly and with few details. Extra-mathematical situations are dealt with only now and then during a course. Normally, no more more than 1-2 lessons are spent on each situation, either in class or with students working individually or in small groups. Issues and problems are formulated from the outset and are supposed to be dealt with by means of recently developed mathematical tools. Extra-mathematical information is either being presupposed or is being provided by the material used, or by the teacher. Students are normally not expected to seek additional information from other sources. The main emphasis of the application and modelling work is on the mathematical aspects of the extra-mathematical situations subject to treatment.

It may be said that if this is a fair description of the average state of applications and modelling in school mathematics there is still a long way to go before applications and modelling can occupy a position in the curriculum which is sensible and balanced and capable of serving well enough the purposes for which applications and modelling should be included at all. That may very well be true, but even if the range and scope of application and modelling activities may be modest compared to what we might wish to see, the most important thing is that they are present, that their rightful citizenship in the curriculum is now being widely accepted. Then it is our next task - a most important one, of course - to improve the quality of the activities.

This task needs not demand an excessive amount of educational ingenuity as far as content and organisation are concerned - when it comes to politics the situation might be different. The large variations in the present state of applications and modelling in school mathematics entails a considerable distance between the forefront and the mainstream. Stimulus and inspiration for improving mainstream curricula can be found in abundance in experiences gained at the forefront.

3. Current trends in applications and modelling

So far we have outlined the place of applications and modelling in school mathematics in static terms. In order to finish the sketch of "where we are" we ought to add some remarks on the dynamical elements of the situation. In a recently published paper by Werner Blum and myself we attempted to identify the most important trends in applications and modelling as of 1988/89. As these trends seem still to prevail it may be sensible to list them here.

* **The spectrum of arguments** for including applications and modelling in mathematics curricula has been **widened**. Earlier, the most frequent arguments were focussed on motivating and assisting the learning of mathematical concepts or topics, or on promoting a utilitarian view of mathematics as a service subject. Today, also arguments emphasising the general formative power of application and modelling activities and their capability of fostering "critical competence" to serve private and social citizenship are encountered, as is the stressing of the role of applications and modelling in generating with students a rich and balanced picture of mathematics as a science and as a field of activity in science and culture.

* Recent years have been witnessing an **increasing globality** of applications and modelling, both in a geographical sense and in terms of the range of curricular levels (primary through tertiary) adopting application and modelling components. It is also part of the increasing globality that the distance between the forefront and the mainstream of application and modelling development is diminishing (although not vanishing), due partly to a strengthening of the mainstream and partly to a decrea-

sing rate of innovation at the forefront.

* An **increasing unification** of three aspects of mathematics which used to be kept somewhat apart, namely applications and modelling, problem solving and links to other subjects, is taking place. This is not to say that we have now reached a stage in which those aspects have been integrated into a homogeneous whole of indiscernible elements, only that the aspects have come closer together and that their mutual relationships have become clearer.

* **The involvement of computers** in application and modelling work is being **extended,** not only *quantitatively* by dissemination of hard- and software to facilitate the handling of models, but also *qualitatively* due to the appearance of interactive tools for model building and model examination.

Since 1988 an additional trend has gained momentum:

* **Assessment** of applications and modelling is attracting an increasing amount of attention of mathematics educators because established modes of assessment and tests are not compatible with the spirit and practice of application and modelling activities.

We shall address this trend further later in this paper.

4. Challenges that ought to be addressed

If we accept the description offered in the previous sections of the present state and current trends of applications and modelling in school mathematics, and if we agree that this shows that the position now occupied by applications and modelling in the average mathematics curriculum is not sensible and balanced, and hence not optimal, our next move will be to ask "what specific challenges in relation to applications and modelling in school mathematics do we need to address in the future?" From my perspective the following are some of the most significant challenges:

(a) What emphasis should be on the **extra-mathematical aspects and phases** of application and modelling work?

(b) To what extent should students be engaged in active and creative **modelling** processes?

(c) What role should **authentic** ("really real") **extra-mathematical situations** have?

(d) How to shape and practise **valid and reliable modes of assessment** of applications and modelling which pay due respect to the charac

teristic content and process components of applications and modelling work?

(e) In what ways will **computers change the conditions** for and practice of application and modelling work?

(f) How to **raise the general level of ambition** in mainstream curricula as regards applications and modelling content and activities?

These are not the only issues which deserve to be addressed when it comes to identifying future directions for development of applications and modelling in school mathematics. A full list of challenges would be rather long, and I have chosen to point to items which to me seem crucial for the overall shaping of applications and modelling in school mathematics.

5. Directions for future development of applications and modelling in school mathematics

In a previous section of this paper it was claimed that average application and modelling work in school mathematics deals with cut-out, simplified and idealised versions of (potentially) realistic situations for which all relevant extra-mathematical information is either presupposed or supplied by written material, or by the teacher. Students are not supposed to seek such information themselves. The main focus of application and modelling work is on its mathematical aspects.

In this section a number of directions for future development will be suggested to improve this state of affairs. Due to space limitations, only a few of the suggestions will be substantiated by a relatively detailed argumentation, whereas the other ones will be corroborated just by terse statements.

In the future, concrete modes and practices should be developed to realise the following objectives:

(a) All essential extra-mathematical aspects and phases of application and modelling work should be represented in school mathematics

When mathematics is activated to deal with an extra-mathematical situation it is always for a **purpose**. Certain aspects of the extra-mathematical situation need to be understood, explained, mastered, controlled, changed, and suchlike, and it is presupposed that mathematics has something to offer to this end.

In order to apply mathematics to an extra-mathematical situation it is necessary to structure the situation, and to specify in precise terms the aspects to be considered and the questions to be answered. In so doing the situation is simplified and idealised, and various explicit and implicit assumptions and conditions are imposed upon it. The result of this "massaging" of the extra-mathematical situation is a **pre-mathematical model** which is sufficiently schematised as to allow for mathematisa
tion.

Mathematisation consists in translating the elements and relations which have been identified as the most important ones in the pre-mathematical model into mathematical objects (belonging to some mathematical universe chosen by the modeller) and relations betwen them. Also the questions asked about the extra-mathematical situation are translated into mathematical questions. All this leads to a **mathematical model** of certain aspects of the situation. Within this model it is possible to work with purely mathematical concepts, methods and theories, and to invoke mathematical results to obtain conclusions about properties of the model.

When the work within the model has been acccomplished its results are **interpreted**, by backward translation, as results concerning the extra-mathematical situation. Then the model has to be **validated**, i.e. its capacity to represent the chosen aspects of the extra-mathematical situation is examined as is the relevance of the model results for the purpose for which it was built. It may happen in consequence of the validation that the model is either rejected or re-built, or that an entirely new model is constructed. It may also happen that the whole thing is given up because it appears that no well-founded model can answer the questions one set out to answer in the first place. The validation process is a most intricate one since it entails all the philosophical, technical and practical difficulties inherent in science.

We may consider the steps outlined above as **phases** of a long and complicated process. We shall call the entire process mathematical **modelling**. Mathematisation is that sub-process of the mathematical modelling process which deals with the translation of a pre-mathematical model into a mathematical model.

The full modelling process often involves a wide variety of activities not mentioned explicitly in the above description of the phases: Obtaining of theoretical and practical information of the extra-mathematical substance; measuring, and collecting data; making assumptions;

formulating conjectures; asking questions; designing experiments; performing statistical investigations such as estimation and testing; acquiring mathematical knowledge and expertise; making a host of decisions regarding which way to follow; and soforth.

Many of these activities and many of the phases in the modelling process inevitably involve extra-mathematical work, even if they imply mathematical considerations as well.

Although this is not the place to present and discuss the arguments for including application and modelling work in school mathematics, let us remind ourselves of the main reason: Mathematics is being applied ever more to a growing spectrum of practical and scientific areas, for better and for worse. For students to become able to cope with this development, in an active as well as in a reflective way, it is not sufficient to simply learn mathematics. They also need to obtain knowledge and skills concerning applications and modelling.

For such knowledge and skills to be well-founded, strong, and flexible, it is essential that students obtain genuine experiences with the **entire modelling process** at each educational level, at least, say, once a year. In addition they should be given opportunities to work in-depth with each of the main phases of the modelling process, including the extra-mathematical ones. In view of the fact that mathematics is often being applied in less than satisfactory ways, I should like to particularly emphasise the validation of models as one particularly important component.

A quick look at the position of applications and modelling in average school mathematics will show that we are far from having met this demand. What is needed of future development in this respect is not only beautiful, convincing papers and conference canvassing, but also a variety of concrete cases to demonstrate how all esssential aspects and phases, mathematical as well as extra-mathematical, of the modelling process can be incorporated into normal school mathenmatics curricula. Unfortunately, the scope of this paper does not allow me to give concrete examples, but quite a few exist.

(b) Once in a while students should be engaged in active modelling work

In principle, students could become acquainted with the essential components of application and modelling work as passive recipients of knowledge transmitted to them through written or oral one-way communication channels such as textbooks or lectures. They could also obtain certain applications and modelling skills by working with tasks in which they are supposed to apply application and modelling schemes previously taught to them in new but structurally identical situations. For instance, if linear (affine) growth models have been introduced to treat car rental with fixed initial costs and an additional rate per day, students may be asked to model a situation in which a plummer having repaired your pipes charges a certain amount for coming to your house plus a rate per working hour.

Such "standardised" activities certainly may have a great educational value. However, in the same way as training to do well-defined, closed exercises in mathematics is important but does not make you capable of performing independent mathematical investigations, or of solving non-routine problems, such activities do not make you a creative constructor or analyst of mathematical models.

For this to be the case it is necessary to have been engaged sufficiently often in performing active and creative modelling processes yourself. In such processes it is neither clear from the outset exactly what to do, nor is it given which mathematical tools to apply to do it.

Generally speaking, students' active and creative modelling is only on the agenda of school mathematics very seldom. To change this situation it is not only necessary to demonstrate how independent modelling could be dealt with in school mathematics. As students' active and creative modelling may easily take them to places which are new to the teacher as well, it is essential to provide pre- and in-service education for teachers to give them enough competence and self-confidence to let them have their students embark on such activities.

(c) Authentic extra-mathematical situations should be included in school mathematics

We define an **authentic** extra-mathematical situation as one which is embedded in a really existing practice or subject area outside mathematics and deals with objects, phenomenae, issues, or problems that are genuine to that area and are recognised as such by people working in it.

It is *not* required in this definition that the extra-mathematical situation must have to do with everyday practical matters from people's lives. It might well deal with highly theoretical matters provided that these actually belong to another subject, e.g. physics or economics. The harmonic oscillator, for instance, is a very idealised and general model of a variety of different oscillating phenomenae, yet it is one of the tools that physicists *do* apply to describe such phenomenae. Also it is not required that the mathematical models involved in dealing with the situation are *valid* or *correct*, whatever that means. What is required is that the situation to which they refer is taken seriously in quarters outside mathematics.

Any treatment of an extra-mathematical situation by applications and modelling means will in some phase focus on purely mathematical objects and relations. This is due to the very process of representing the situation mathematically - and to the very point in so doing. The elements of selection, idealisation, simplification etc. that are inherent in this process do not in themselves take the authenticity away from the situation. What matters here is whether the tailoring has been made *outside* the practice or subject area to which the situation belongs, and *prior* to the inclusion of the situation in mathematics teaching. An originally authentic extra-mathematical situation may lose its authenticity in mathematics teaching. This will happen if the tailored version presented in the classroom is either no longer familiar to people in the original area, or has not resulted from modifications made *only in* the mathematics teaching, whether as included in the material or during classroom activities.

Authentic extra-mathematical situations, thus defined, are not in focus of application and modelling activities in school mathematics very often. It is also *frequently the case that it is not possible to tell whether or not an application and modelling situation as presented to students is authentic or not*. Here is an example from a Danish public written examination paper (sat at the end of grade 12) given in 1977:

With healthy human beings the concentration of blood sugar is close to 100 mg per 100 ml blood. Injection of a certain dose of insuline changes the concentration of blood sugar. The concentration, measured in mg per 100 ml, is a function, f, of the time, x (measured in hours), which has elapsed after injection. We assume that

$$f(x) = 100 + 111(\exp(-4x) - \exp(-0.8x)).$$

(...)
Determine the point of time at which the concentration of blood sugar has the fastest increase.

There is little doubt that this describes an extra-mathematical situation. But based on the information given (and no more was available to the students who sat the paper) we are not able to tell to which extent the situation is an authentic one.

We do not know whether the average concentration of blood sugar indicated is authentic, or if the specific function f is actually suggested or used, exactly as it stands with parameters etc., by physicians to describe the effect of insuline injections. It might be, for instance, that a similar but more complicated function, or one with less simple parameters, is in use. Or it might be that the committee of matematics teachers who formulated the task have constructed or modified the model themselves. We also do not know whether the question asked at the end represents a genuine medical problem, and if so we don't know why. Maybe it would be medically more relevant to ask when the blood sugar concentration is maximal, or how many hours after the insuline injection it would take before the blood sugar concentration gets back to normal (if ever).

However, the presentation quoted above pretends but does not assert that the task is authentic. I suspect that we have to do with a fairly modified version of an authentic model, but that the question asked in the task has hardly any medical relevance. The purpose of the task seems to be to present an extra-mathematical situation, *similar* to an authentic one, which in disguise leads to the activation of recently developed mathematical tools (*in casu* extreme problems tackled by differential calculus). So students are expected to do two things: (1) remove the disguise, and (2) perform a rather standard mathematical exercise.

I believe it is important that students are confronted with authentic extra-mathematical situations in school mathematics every now and then. Otherwise all application and modelling work has to do with "as if" situations, even if some of them may be similar to authentic ones. In that way many students are likely to get the impression that application and modelling situations only serve to disguise pure mathematics in pseudo-real clothes. This leads to the perception that applications and modelling is a sort of *game* - a game which may be good fun, and perhaps even be similar to real application and modelling activities as they exist outside mathematics teaching. It leads, furthermore, to the perception that school mathematics is not powerful enough to treat authentic extra-mathematical situations and problems. And from there it only takes a small step to arrive at the perception that school mathematics is useless.

One point seems to me to be particularly important: The mathematics curriculum should not be confined to including authentic sitations which can be dealt with in 1-2 lessons. For only very limited or truncated authentic situations can be treated satisfactorily within that time constraint. Larger authentic application and modelling situations that may take several weeks, perhaps, should be included as well if we want mathematics teaching to provide a balanced representation of what it implies to subject authentic extra-mathematical situations to mathematical treatment.

Against this background a lot of work needs to be done to find ways to incorporate authentic extra-mathematical situations, smaller as well as larger, in school mathematics. This involves identification of suitable examples and didactical consideration of how to include them in the teaching of mathematics. Both tasks are non-trivial, and the more so the more sophisticated mathematics is involved. There are things being done in this direction in various places in the world, but much more needs to be done.

I am not claiming that every extra-mathematical situation dealt with by applications and modelling means has to be authentic. There is much good educational value concerning applications and modelling to be found also in purely artificial situations. I am also not claiming that authenticity is the only factor of significance as regards application and modelling work of quality. An example may serve to illustrate this point.

In my country it is not unusual that mathematics textbooks for upper secondary schools describe in considerable detail the Carbon-14 method to date archeological objects containing material from dead organisms, mostly plants. This is done by applying an exponential model of radioactive decay to C-14 which has a halving time of ca. 5700 years. As the ratio between C-12 (non-radioactive) and C-14 in living organisms is (assumed to be) approximately constant over time, it is possible to measure the fraction of C-14 which still remains in the sample. On that basis the specific exponential function involved can be determined. This enables us to calculate the time which has elapsed since the death of the organism considered.

With this model archaelogists' attempts to date real objects can be followed and reproduced by students. As far as textbook presentation, student work, classroom activities and soforth are concerned, the Carbon-14 method can be taught exactly as completely artificial application and modelling situations, or topics in pure mathematics for that matter, are taught. With authentic extra-mathematical situations (cf. also the harmonic oscillator) students may be equally passive recipients of teaching as with artifical situations. However, the fact that this method is *actually used* by archaeologists and that students could do the calculations themselves *does* make a difference. My point here is to emphasise that there is more to be discussed than authenticity alone.

(d) Current modes of assessment and testing in mathematics education are inappropriate to applications and modelling, new modes have to be devised

This is not the place go much into detail with the issue of assessment as I have recently addressed this topic at length elsewhere (cf. [3]). I shall confine myself to mentioning a few major points.

In a curriculum which makes use of formalised assessment, as most curricula do, components which are not subject to assessment are not taken seriously. Hence application and modelling work needs to be assessed in spite of the reservations one might have about the possibility of assessing such highly intricate and complicated activities. Our task is to answer the question: how to assess applications and modelling without destroying it?

The content and organisation of assessment should be shaped in such a way that they pay due respect to the purpose and nature of application and modelling work in the curriculum, and not the other way round.

We ought to occasionally assess students' work with the entire process of applications and modelling in all its phases. In particular we should validate the scientific quality of models constructed by students, and assess students' validation of models and applications.

Assessment should take place for tasks and activities which are appropriate for letting application and modelling work unfold. Most current tasks and activities uased in assessment are inappropriate, because they do not make room either for essential content components or essential process components of applications and modelling. Especially, this holds for tasks and activities exercised under usual test and exam conditions with time constraints.

There *is* a variety of tasks and activities through which application and modelling work can be suitably assessed: extended investigations, student portfolios, and projects. But they are incompatible with most of the traditional requirements usually enforced on assessment in mathematics education. According to those requirements assessment should: take place under test conditions with time constraints; be inexpensive in terms of the effort spent; be implementable on a large scale; not allow for disagreement between assessors; allow for easy marking of students' achievement.

We should invest effort in explaining all this to curriculum authorities, teachers, parents, etc., and to convince them that change is needed. While waiting for this to work we should invest more ingenuity in finding new modes to assess applications and modelling that can circumvent the traditional difficulties and restrictions.

(e) Computers call for more emphasis on mathematical reasoning and "analysis"

Computer hard- and software continue to offer new opportunities and new challenges to mathematics teaching in general and to applications and modelling in particular. There is every reason to believe that this will remain to be the case also in the future. Although it is likely that the future will bring us new and hitherto unforeseen computer products, some of which may differ qualitatively from what we know today, I believe that we are, already today, in a position to identify the major challenges computers present in relation to applications and modelling in mathematics teaching and to discuss how these challenges ought to be met.

Let us review very briefly what computerware is at the disposal of applications and modelling in school mathematics at the moment - in countries which can afford to buy it.

As far as hardware is concerned the spectrum ranges from cheap non-programmable pocket calculators, over programmable ones and portable computers, to personal computers and workstations, perhaps as part of a network. It is likely that in the near future CD-roms and interactive video will be generally affordable as well. But so far they are not available for school mathematics purposes on a large international scale.

Software that is relevant to application and modelling work falls within four different categories:

- Software which can do **numerical computations** and **calculations**, not only with numbers, but also with functions, and can do **numerical analysis** (solve linear and non-linear equations, including ODEs and PDEs, solve linear programming problems, invert matrices, calculate eigen-values etc.).

- **Graphical software** which can display all sorts of tables, diagrams, solutions to equations and inequalities, graphs of functions, geometrical objects, flow-charts etc., show the effect of changing parameters, display low-dimensional sections of high-dimensional objects, and soforth.

- Software which can perform **symbolic calculations** within an algorithmically organised universe. Examples: reduction of algebraic expressions, determination of indefinite integrals, solution of standard classes of difference and differential equations written in algebraic form.

- Software which offer specific **tools to facilitate modelling**, such as System Dynamics-type programmes, modelling with differential equations, programmel designed to explore the behaviour of mathematical objects arising in application and modelling situations, and symbolic specification languages. A large and important class of software in this category is software which can perform a multitude of standard **statistical analyses** such as computing descriptors, carrying out estimation of parameters, testing of hypotheses, performing variance analysis, regression analysis, curve-fitting, and factor analysis. Much software in this category not only makes the calculations involved in a statistical problem but also takes care of the actual analysis, for instance by identifying the explaining variables in the problem. Most of these tools in this category are made by combining selected parts of software belonging to the three first-mentioned categories.

The above categories of opportunities at the disposal to school mathematics evidently make it possible to facilitate the handling of application and modelling situations. Tedious computations and calculations no longer represent unsurmountable hurdles for progress. In this way it makes sense to build models which are thought to provide a satisfactory covering of the situation modelled but which cannot be tackled by "clasiccal", analytic means but can be tackled with computerised means. It also becomes possible to explore and discover consequences of assumptions and the effects of changing them, the effect of varying parameters, adding terms etc., and performing simulations. This makes *experimentation with and within models* possible. In this way it is possible to gain a wide variety of experiences and thus a much stronger and elaborate feeling for what can happen. This helps one to formulate hypotheses etc.

In my opinion there is no doubt that all this *does* open very valuable avenues to application and modelling work. But there are also dangers involved.

The problem is that with computers we can **do** things that we **do not understand.** Complicated models can be cracked with brute force and spectacular results can be obtained. But quite often we do not know why the results are as they are. We do not know which features are fundamental and which are incidental. We do not know to which degree of accuracy the results are correct. We do not know whether other solutions exist which may have been ignored due to the processes applied etc. It may be tempting to conclude that now that computers can do a lot of things for us that before we could only with considerable difficulty we should skip dealing with those aspects in our mathematics teaching. My conclusion is the opposite: The more computers can do for us the more important becomes the necessity to analyse what is going on. Instead of reducing the amount of mathematics which we need to learn it *increases* it. I am not saying that there are no old routines that can be dispensed with. There are, namely those routines connected to the learning and performance of *isolated*, singular algorithms, for instance interpolation techniques with logarithms, certain procedures for extracting square roots and suchlike. The advent and dissemination of pocket calculators have not made arithmetic obsolete, because pocket calculators do not tell us when to do a certain operation, and because the fact that it is easy to press the wrong button makes gross calculations important. So to actually being able to utilise the free space given to us by pocket calculators we have to be able to do the calculations ourselves. It is much the same with computers. If we are to remain the masters and not the servants in our relationship with the computers in applications and modelling we have to know the foundation of the work they do for us. Otherwise we don't know when to rely on them and when to be skeptical. When we know that we can leave things to the computer.

Computers should offer us *prolongations of our limbs not substitutes for them.*

So, as far as future developments in this areas are concerned we should do much more to investigate not only the new opportunities provided by computers but also the pitfalls and limits of those opportunities. This is a call for concrete research. A bank of different cases which demonstrate the goods and the evils of computers in relation to applications and modelling is much needed. We should begin to establish such a bank right away.

(f) The general level of ambition in mainstream curricula ought to be raised

It is not important in itself to reduce the distance between the forefront and the mainstream of applications and modelling in mathematics teaching - after all we are not running a competition. The important thing is to make the mainstream be satisfactory in its own right. New developments at the forefront can serve to make this happen. The role of vanguard is to gain experiences and insights on behalf of the rest of us. So, progress made in relation to the above five new directions of development will also serve to help raising the level of ambition of application and modelling work in mainstream school mathematics.

If we activate some of the findings in items (a)-(e) towards the present item we get some indications of how to raise the general level of ambition:

* We should be able and prepared to show that much of what is done i mainstream applications and modelling *is* too poor.

* We should procure a variety of specific examples to demonstrate that more ambitious things are *accessible* within the framework of *normal* curricula.

* We should devise ways to stimulate teachers to exchange views, experiences and materials, for instance through in-service courses.

* In curricula which are centrally controlled we should include explicit requirements regarding the presence of applications and modelling.

* We should change assessment so as to make it possible to pay full respect to application and modelling work.

* We should always make it clear that applications and modelling should not be there for their own sake. They should be included in school mathematics curricula to make school mathematics much better at serving the educational purposes **it** is there for.

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Vejleder: Anders Madsen

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3/78 "OPGAVESAMLING", breddekursus i fysik.
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Projektrapport af: Lasse Rasmussen .
Vejleder: Anders Madsen.

10/79 "TERMODYNAMIK I GYMNASIET".
Projektrapport af: Jan Christensen og Jeanne Mortensen,
Vejledere: Karin Beyer og Peder Voetmann Christiansen.

11/79 "STATISTISKE MATERIALER".
Af: Jørgen Larsen.

12/79 "LINEÆRE DIFFERENTIALLIGNINGER OG DIFFERENTIALLIGNINGSSYSTEMER".
Af: Mogens Brun Heefelt.
Nr. 12 er udgået.

13/79 "CAVENDISH'S FORSØG I GYMNASIET".
Projektrapport af: Gert Kreinøe.
Vejleder: Albert Chr. Paulsen.

14/79 "BOOKS ABOUT MATHEMATICS: History, Philosophy, Education, Models, System Theory, and Works of".
Af: Else Høyrup.
Nr. 14 er p.t. udgået.

15/79 "STRUKTUREL STABILITET OG KATASTROFER i systemer i og udenfor termodynamisk ligevægt".
Specialeopgave af: Leif S. Striegler.
Vejleder: Peder Voetmann Ckristiansen.

16/79 "STATISTIK I KRÆFTFORSKNINGEN".
Projektrapport af: Michael Olsen og Jørn Jensen.
Vejleder: Jørgen Larsen.

17/79 "AT SPØRGE OG AT SVARE i fysikundervisningen".
Af: Albert Christian Paulsen.

18/79 "MATHEMATICS AND THE REAL WORLD", Proceedings af an International Workshop, Roskilde University Centre, Denmark, 1978.
Preprint.
Af: Bernhelm Booss og Mogens Niss (eds.)

19/79 "GEOMETRI, SKOLE OG VIRKELIGHED".
Projektrapport af: Tom J. Andersen, Tommy R. Andersen og Per H.H. Larsen.
Vejleder: Mogens Niss.

20/79 "STATISTISKE MODELLER TIL BESTEMMELSE AF SIKRE DOSER FOR CARCINOGENE STOFFER".
Projektrapport af: Michael Olsen og Jørn Jensen.
Vejleder: Jørgen Larsen

21/79 "KONTROL I GYMNASIET-FORMÅL OG KONSEKVENSER".
Projektrapport af: Crilles Bacher, Per S.Jensen, Preben Jensen og Torben Nysteen.

22/79 "SEMIOTIK OG SYSTEMEGENSKABER (1)".
1-port lineært response og støj i fysikken.
Af: Peder Voetmann Christiansen.

23/79 "ON THE HISTORY AF EARLY WAVE MECHANICS - with special emphasis on the role af realitivity".
Af: Helge Kragh.

24/80 a+b "MATEMATIKOPFATTELSER HOS 2.G'ERE".
1. En analyse. 2. Interviewmateriale.
Projektrapport af: Jan Christensen og Knud Lindhardt Rasmussen.
Vejleder: Mogens Niss.

25/80 "EKSAMENSOPGAVER", Dybdemodulet/fysik 1974-79.

26/80 "OM MATEMATISKE MODELLER".
En projektrapport og to artikler.
Af: Jens Højgaard Jensen m.fl.

27/80 "METHODOLOGY AND PHILOSOPHY AF SCIENCE IN PAUL DIRAC's PHYSICS".
Af: Helge Kragh.

28/80 "DIELEKTRISK RELAXATION - et forslag til en ny model bygget på væskernes viscoelastiske egenskaber".
Projektrapport af: Gert Kreinøe.
Vejleder: Niels Boye Olsen.

29/80 "ODIN - undervisningsmateriale til et kursus i differentialligningsmodeller".
Projektrapport af: Tommy R. Andersen, Per H.H. Larsen og Peter H. Lassen.
Vejleder: Mogens Brun Heefelt.

30/80 "FUSIONSENERGIEN - - - ATOMSAMFUNDETS ENDESTATION".
Af: Oluf Danielsen.
Nr. 30 er udgået.

31/80 "VIDENSKABSTEORETISKE PROBLEMER VED UNDERVISNINGSSYSTEMER BASERET PÅ MÆNGDELÆRE".
Projektrapport af: Troels Lange og Jørgen Karrebæk.
Vejleder: Stig Andur Pedersen.
Nr. 31 er p.t. udgået.

32/80 "POLYMERE STOFFERS VISCOELASTISKE EGENSKABER - BELYST VED HJÆLP AF MEKANISKE IMPEDANSMÅLINGER MÖSSBAUEREFFEKTMÅLINGER".
Projektrapport af: Crilles Bacher og Preben Jensen.
Vejledere: Niels Boye Olsen og Peder Voetmann Christiansen.

33/80 "KONSTITUERING AF FAG INDEN FOR TEKNISK - NATURVIDENSKABELIGE UDDANNELSER. I-II".
Af: Arne Jakobsen.

34/80 "ENVIRONMENTAL IMPACT AF WIND ENERGY UTILIZATION".
ENERGY SERIES NO. I.
Af: Bent Sørensen
Nr. 34 er udgået.

35/80 "HISTORISKE STUDIER I DEN NYERE ATOMFYSIKS UDVIKLING".
Af: Helge Kragh.

36/80 "HVAD ER MENINGEN MED MATEMATIKUNDERVISNINGEN?".
Fire artikler.
Af: Mogens Niss.

37/80 "RENEWABLE ENERGY AND ENERGY STORAGE".
ENERGY SERIES NO. 2.
Af: Bent Sørensen.

38/81 "TIL EN HISTORIETEORI OM NATURERKENDELSE, TEKNOLOGI OG SAMFUND".
Projektrapport af: Erik Gade, Hans Hedal, Henrik Lau og Finn Physant.
Vejledere: Stig Andur Pedersen, Helge Kragh og Ib Thiersen.
Nr. 38 er p.t. udgået.

39/81 "TIL KRITIKKEN AF VÆKSTØKONOMIEN".
Af: Jens Højgaard Jensen.

40/81 "TELEKOMMUNIKATION I DANMARK - oplæg til en teknologivurdering".
Projektrapport af: Arne Jørgensen, Bruno Petersen og Jan Vedde.
Vejleder: Per Nørgaard.

41/81 "PLANNING AND POLICY CONSIDERATIONS RELATED TO THE INTRODUCTION OF RENEWABLE ENERGY SOURCES INTO ENERGY SUPPLY SYSTEMS".
ENERGY SERIES NO. 3.
Af: Bent Sørensen.

42/81 "VIDENSKAB TEORI SAMFUND - En introduktion til materialistiske videnskabsopfattelser".
Af: Helge Kragh og Stig Andur Pedersen.

43/81 1."COMPARATIVE RISK ASSESSMENT OF TOTAL ENERGY SYSTEMS".
2."ADVANTAGES AND DISADVANTAGES OF DECENTRALIZATION".
ENERGY SERIES NO. 4.
Af: Bent Sørensen.

44/81 "HISTORISKE UNDERSØGELSER AF DE EKSPERIMENTELLE FORUDSÆTNINGER FOR RUTHERFORDS ATOMMODEL".
Projektrapport af: Niels Thor Nielsen.
Vejleder: Bent C. Jørgensen.

45/82 Er aldrig udkommet.

46/82 "EKSEMPLARISK UNDERVISNING OG FYSISK ERKENDESE-
1+11 ILLUSTRERET VED TO EKSEMPLER".
Projektrapport af: Torben O.Olsen, Lasse Rasmussen og Niels Dreyer Sørensen.
Vejleder: Bent C. Jørgensen.

47/82 "BARSEBÄCK OG DET VÆRST OFFICIELT-TÆNKELIGE UHELD".
ENERGY SERIES NO. 5.
Af: Bent Sørensen.

48/82 "EN UNDERSØGELSE AF MATEMATIKUNDERVISNINGEN PÅ ADGANGSKURSUS TIL KØBENHAVNS TEKNIKUM".
Projektrapport af: Lis Eilertzen, Jørgen Karrebæk, Troels Lange, Preben Nørregaard, Lissi Pedesen, Laust Rishøj, Lill Røn og Isac Showiki.
Vejleder: Mogens Niss.

49/82 "ANALYSE AF MULTISPEKTRALE SATELLITBILLEDER".
Projektrapport af: Preben Nørregaard.
Vejledere: Jørgen Larsen og Rasmus Ole Rasmussen.

50/82 "HERSLEV - MULIGHEDER FOR VEDVARENDE ENERGI I EN LANDSBY".
ENERGY SERIES NO. 6.
Rapport af: Bent Christensen, Bent Hove Jensen, Dennis B. Møller, Bjarne Laursen, Bjarne Lillethorup og Jacob Mørch Pedersen.
Vejleder: Bent Sørensen.

51/82 "HVAD KAN DER GØRES FOR AT AFHJÆLPE PIGERS BLOKERING OVERFOR MATEMATIK ?"
Projektrapport af: Lis Eilertzen, Lissi Pedersen, Lill Røn og Susanne Stender.

52/82 "DESUSPENSION OF SPLITTING ELLIPTIC SYMBOLS".
Af: Bernhelm Booss og Krzysztof Wojciechowski.

53/82 "THE CONSTITUTION OF SUBJECTS IN ENGINEERING EDUCATION".
Af: Arne Jacobsen og Stig Andur Pedersen.

54/82 "FUTURES RESEARCH" - A Philosophical Analysis of Its Subject-Matter and Methods.
Af: Stig Andur Pedersen og Johannes Witt-Hansen.

55/82 "MATEMATISKE MODELLER" - Litteratur på Roskilde Universitetsbibliotek.
En biografi.
Af: Else Høyrup.

Vedr. tekst nr. 55/82 se også tekst nr. 62/83.

56/82 "EN - TO - MANGE" -
En undersøgelse af matematisk økologi.
Projektrapport af: Troels Lange.
Vejleder: Anders Madsen.

57/83 "ASPECT EKSPERIMENTET"-
Skjulte variable i kvantemekanikken?
Projektrapport af: Tom Juul Andersen.
Vejleder: Peder Voetmann Christiansen.
Nr. 57 er udgået.

58/83 "MATEMATISKE VANDRINGER" - Modelbetragtninger over spredning af dyr mellem småbiotoper i agerlandet.
Projektrapport af: Per Hammershøj Jensen og Lene Vagn Rasmussen.
Vejleder: Jørgen Larsen.

59/83 "THE METHODOLOGY OF ENERGY PLANNING".
ENERGY SERIES NO. 7.
Af: Bent Sørensen.

60/83 "MATEMATISK MODEKSPERTISE"- et eksempel.
Projektrapport af: Erik O. Gade, Jørgen Karrebæk og Preben Nørregaard.
Vejleder: Anders Madsen.

61/83 "FYSIKS IDEOLOGISKE FUNKTION, SOM ET EKSEMPEL PÅ EN NATURVIDENSKAB - HISTORISK SET".
Projektrapport af: Annette Post Nielsen.
Vejledere: Jens Høyrup, Jens Højgaard Jensen og Jørgen Vogelius.

62/83 "MATEMATISKE MODELLER" - Litteratur på Roskilde Universitetsbibliotek.
En biografi 2. rev. udgave.
Af: Else Høyrup.

63/83 "GREATING ENERGY FUTURES:A SHORT GUIDE TO ENERGY PLANNING".
ENERGY SERIES No. 8.
Af: David Crossley og Bent Sørensen.

64/83 "VON MATEMATIK UND KRIEG".
Af: Berhelm Booss og Jens Høyrup.

65/83 "ANVENDT MATEMATIK - TEORI ELLER PRAKSIS".
Projektrapport af: Per Hedegård Andersen, Kirsten Habekost, Carsten Holst-Jensen, Annelise von Moos, Else Marie Pedersen og Erling Møller Pedersen.
Vejledere: Bernhelm Booss og Klaus Grünbaum.

66/83 "MATEMATISKE MODELLER FOR PERIODISK SELEKTION I ESCHERICHIA COLI".
Projektrapport af: Hanne Lisbet Andersen, Ole Richard Jensen og Klavs Frisdahl.
Vejledere: Jørgen Larsen og Anders Hede Madsen.

67/83 "ELEPSOIDE METODEN - EN NY METODE TIL LINEÆR PROGRAMMERING?"
Projektrapport af: Lone Biilmann og Lars Boye.
Vejleder: Mogens Brun Heefelt.

68/83 "STOKASTISKE MODELLER I POPULATIONSGENETIK"
- til kritikken af teoriladede modeller.
Projektrapport af: Lise Odgård Gade, Susanne Hansen, Michael Hviid og Frank Mølgård Olsen.
Vejleder: Jørgen Larsen.

69/83 "ELEVFORUDSÆTNINGER I FYSIK"
- en test i 1.g med kommentarer.
Af: Albert C. Paulsen.

70/83 "INDLÆRINGS - OG FORMIDLINGSPROBLEMER I MATEMATIK PÅ VOKSENUNDERVISNINGSNIVEAU".
Projektrapport af: Hanne Lisbet Andersen, Torben J. Andreasen, Svend Åge Houmann, Helle Glerup Jensen, Keld Fl. Nielsen, Lene Vagn Rasmussen.
Vejleder: Klaus Grünbaum og Anders Hede Madsen.

71/83 "PIGER OG FYSIK"
- et problem og en udfordring for skolen?
Af: Karin Beyer, Sussanne Blegaa, Birthe Olsen, Jette Reich og Mette Vedelsby.

72/83 "VERDEN IFØLGE PEIRCE" - to metafysiske essays, om og af C.S Peirce.
Af: Peder Voetmann Christiansen.

73/83 ""EN ENERGIANALYSE AF LANDBRUG"
- økologisk contra traditionelt.
ENERGY SERIES NO. 9
Specialeopgave i fysik af: Bent Hove Jensen.
Vejleder: Bent Sørensen.

74/84 "MINIATURISERING AF MIKROELEKTRONIK" - om videnskabeliggjort teknologi og nytten af at lære fysik.
Projektrapport af: Bodil Harder og Linda Szkotak Jensen.
Vejledere: Jens Højgaard Jensen og Bent C. Jørgensen.

75/84 "MATEMATIKUNDERVISNINGEN I FREMTIDENS GYMNASIUM"
- Case: Lineær programmering.
Projektrapport af: Morten Blomhøj, Klavs Frisdahl og Frank Mølgaard Olsen.
Vejledere: Mogens Brun Heefelt og Jens Bjørneboe.

76/84 "KERNEKRAFT I DANMARK?" - Et høringssvar indkaldt af miljøministeriet, med kritik af miljøstyrelsens rapporter af 15. marts 1984.
ENERGY SERIES No. 10
Af: Niels Boye Olsen og Bent Sørensen.

77/84 "POLITISKE INDEKS - FUP ELLER FAKTA?"
Opinionsundersøgelser belyst ved statistiske modeller.
Projektrapport af: Svend Åge Houmann, Keld Nielsen og Susanne Stender.
Vejledere: Jørgen Larsen og Jens Bjørneboe.

78/84 "JÆVNSTRØMSLEDNINGSEVNE OG GITTERSTRUKTUT I AMORFT GERMANIUM".
Specialrapport af: Hans Hedal, Frank C. Ludvigsen og Finn C. Physant.
Vejleder: Niels Boye Olsen.

79/84 "MATEMATIK OG ALMENDANNELSE".
Projektrapport af: Henrik Coster, Mikael Wennerberg Johansen, Povl Kattler, Birgitte Lydholm og Morten Overgaard Nielsen.
Vejleder: Bernhelm Booss.

80/84 "KURSUSMATERIALE TIL MATEMATIK B".
Af: Mogens Brun Heefelt.

81/84 "FREKVENSAFHÆNGIG LEDNINGSEVNE I AMORFT GERMANIUM".
Specialerapport af: Jørgen Wind Petersen og Jan Christensen.
Vejleder: Niels Boye Olsen.

82/84 "MATEMATIK - OG FYSIKUNDERVISNINGEN I DET AUTOMATISEREDE SAMFUND".
Rapport fra et seminar afholdt i Hvidovre 25-27 april 1983.
Red.: Jens Højgaard Jensen, Bent C. Jørgensen og Mogens Niss.

83/84 "ON THE QUANTIFICATION OF SECURITY":
PEACE RESEARCH SERIES NO. I
Af: Bent Sørensen
nr. 83 er p.t. udgået

84/84 "NOGLE ARTIKLER OM MATEMATIK, FYSIK OG ALMENDANNELSE".
Af: Jens Højgaard Jensen, Mogens Niss m. fl.

85/84 "CENTRIFUGALREGULATORER OG MATEMATIK".
Specialerapport af: Per Hedegård Andersen, Carsten Holst-Jensen, Else Marie Pedersen og Erling Møller Pedersen.
Vejleder: Stig Andur Pedersen.

86/84 "SECURITY IMPLICATIONS OF ALTERNATIVE DEFENSE OPTIONS FOR WESTERN EUROPE".
PEACE RESEARCH SERIES NO. 2
Af: Bent Sørensen.

87/84 "A SIMPLE MODEL OF AC HOPPING CONDUCTIVITY IN DISORDERED SOLIDS".
Af: Jeppe C. Dyre.

88/84 "RISE, FALL AND RESURRECTION OF INFINITESIMALS".
Af: Detlef Laugwitz.

89/84 "FJERNVARMEOPTIMERING".
Af: Bjarne Lillethorup og Jacob Mørch Pedersen.

90/84 "ENERGI I 1.G - EN TEORI FOR TILRETTELÆGGELSE".
Af: Albert Chr. Paulsen.

91/85 "KVANTETEORI FOR GYMNASIET".
1. Lærervejledning
Projektrapport af: Biger Lundgren, Henning Sten Hansen og John Johansson.
Vejleder: Torsten Meyer.

92/85 "KVANTETEORI FOR GYMNASIET".
2. Materiale
Projektrapport af: Biger Lundgren, Henning Sten Hansen og John Johansson.
Vejleder: Torsten Meyer.

93/85 "THE SEMIOTICS OF QUANTUM - NON - LOCALITY".
Af: Peder Voetmann Christiansen.

94/85 "TREENIGHEDEN BOURBAKI - generalen, matematikeren og ånden".
Projektrapport af: Morten Blomhøj, Klavs Frisdahl og Frank M. Olsen.
Vejleder: Mogens Niss.

95/85 "AN ALTERNATIV DEFENSE PLAN FOR WESTERN EUROPE".
PEACE RESEARCH SERIES NO. 3
Af: Bent Sørensen

96/85 "ASPEKTER VED KRAFTVARMEFORSYNING".
Af: Bjarne Lilletorup.
Vejleder: Bent Sørensen.

97/85 "ON THE PHYSICS OF A.C. HOPPING CONDUCTIVITY".
Af: Jeppe C. Dyre.

98/85 "VALGMULIGHEDER I INFORMATIONSALDEREN".
Af: Bent Sørensen.

99/85 "Der er langt fra Q til R".
Projektrapport af: Niels Jørgensen og Mikael Klintorp.
Vejleder: Stig Andur Pedersen.

100/85 "TALSYSTEMETS OPBYGNING".
Af: Mogens Niss.

101/85 "EXTENDED MOMENTUM THEORY FOR WINDMILLS IN PERTURBATIVE FORM".
Af: Ganesh Sengupta.

102/85 OPSTILLING OG ANALYSE AF MATEMATISKE MODELLER, BELYST VED MODELLER OVER KØERS FODEROPTAGELSE OG - OMSÆTNING".
Projektrapport af: Lis Eilertzen, Kirsten Habekost, Lill Røn og Susanne Stender.
Vejleder: Klaus Grünbaum.

103/85 "ØDSLE KOLDKRIGERE OG VIDENSKABENS LYSE IDEER".
Projektrapport af: Niels Ole Dam og Kurt Jensen.
Vejleder: Bent Sørensen.

104/85 "ANALOGREGNEMASKINEN OG LORENZLIGNINGER".
Af: Jens Jæger.

105/85 "THE FREQUENCY DEPENDENCE OF THE SPECIFIC HEAT AF THE GLASS REANSITION".
Af: Tage Christensen.

"A SIMPLE MODEL AF AC HOPPING CONDUCTIVITY".
Af: Jeppe C. Dyre.
Contributions to the Third International Conference on the Structure of Non - Crystalline Materials held in Grenoble July 1985.

106/85 "QUANTUM THEORY OF EXTENDED PARTICLES".
Af: Bent Sørensen.

107/85 "EN MYG GØR INGEN EPIDEMI".
- flodblindhed som eksempel på matematisk modellering af et epidemiologisk problem.
Projektrapport af: Per Hedegård Andersen, Lars Boye, Carsten Holst Jensen, Else Marie Pedersen og Erling Møller Pedersen.
Vejleder: Jesper Larsen.

108/85 "APPLICATIONS AND MODELLING IN THE MATEMATICS CURRICULUM" - state and trends -
Af: Mogens Niss.

109/85 "COX I STUDIETIDEN" - Cox's regressionsmodel anvendt på studenteroplysninger fra RUC.
Projektrapport af: Mikael Wennerberg Johansen, Poul Katler og Torben J. Andreasen.
Vejleder: Jørgen Larsen.

110/85 "PLANNING FOR SECURITY".
Af: Bent Sørensen.

111/85 JORDEN RUNDT PÅ FLADE KORT".
Projektrapport af: Birgit Andresen, Beatriz Quinones og Jimmy Staal.
Vejleder: Mogens Niss.

112/85 "VIDENSKABELIGGØRELSE AF DANSK TEKNOLOGISK INNOVATION FREM TIL 1950 - BELYST VED EKSEMPLER".
Projektrapport af: Erik Odgaard Gade, Hans Hedal, Frank C. Ludvigsen, Annette Post Nielsen og Finn Physant.
Vejleder: Claus Bryld og Bent C. Jørgensen.

113/85 "DESUSPENSION OF SPLITTING ELLIPTIC SYMBOLS 11".
Af: Bernhelm Booss og Krzysztof Wojciechowski.

114/85 "ANVENDELSE AF GRAFISKE METODER TIL ANALYSE AF KONTIGENSTABELLER".
Projektrapport af: Lone Biilmann, Ole R. Jensen og Arne-Lise von Moos.
Vejleder: Jørgen Larsen.

115/85 "MATEMATIKKENS UDVIKLING OP TIL RENÆSSANCEN".
Af: Mogens Niss.

116/85 "A PHENOMENOLOGICAL MODEL FOR THE MEYER-NELDEL RULE".
Af: Jeppe C. Dyre.

117/85 "KRAFT & FJERNVARMEOPTIMERING"
Af: Jacob Mørch Pedersen.
Vejleder: Bent Sørensen

118/85 "TILFÆLDIGHEDEN OG NØDVENDIGHEDEN IFØLGE PEIRCE OG FYSIKKEN".
Af: Peder Voetmann Christiansen

119/86 "DET ER GANSKE VIST - - EUKLIDS FEMTE POSTULAT KUNNE NOK SKABE RØRE I ANDEDAMMEN".
Af: Iben Maj Christiansen
Vejleder: Mogens Niss.

120/86 "ET ANTAL STATISTISKE STANDARDMODELLER".
Af: Jørgen Larsen

121/86 "SIMULATION I KONTINUERT TID".
Af: Peder Voetmann Christiansen.

122/86 "ON THE MECHANISM OF GLASS IONIC CONDUCTIVITY".
Af: Jeppe C. Dyre.

123/86 "GYMNASIEFYSIKKEN OG DEN STORE VERDEN".
Fysiklærerforeningen, IMFUFA, RUC.

124/86 "OPGAVESAMLING I MATEMATIK".
Samtlige opgaver stillet i tiden 1974-jan. 1986.

125/86 "UVBY,β - systemet - en effektiv fotometrisk spektralklassifikation af B-,A- og F-stjerner".
Projektrapport af: Birger Lundgren.

126/86 "OM UDVIKLINGEN AF DEN SPECIELLE RELATIVITETSTEORI".
Projektrapport af: Lise Odgaard & Linda Szkotak Jensen
Vejledere: Karin Beyer & Stig Andur Pedersen.

127/86 "GALOIS' BIDRAG TIL UDVIKLINGEN AF DEN ABSTRAKTE ALGEBRA".
Projektrapport af: Pernille Sand, Heine Larsen & Lars Frandsen.
Vejleder: Mogens Niss.

128/86 "SMÅKRYB" - om ikke-standard analyse.
Projektrapport af: Niels Jørgensen & Mikael Klintorp.
Vejleder: Jeppe Dyre.

129/86 "PHYSICS IN SOCIETY"
Lecture Notes 1983 (1986)
Af: Bent Sørensen

130/86 "Studies in Wind Power"
Af: Bent Sørensen

131/86 "FYSIK OG SAMFUND" - Et integreret fysik/historieprojekt om naturanskuelsens historiske udvikling og dens samfundsmæssige betingethed.
Projektrapport af: Jakob Heckscher, Søren Brønd, Andy Wierød.
Vejledere: Jens Høyrup, Jørgen Vogelius, Jens Højgaard Jensen.

132/86 "FYSIK OG DANNELSE"
Projektrapport af: Søren Brønd, Andy Wierød.
Vejledere: Karin Beyer, Jørgen Vogelius.

133/86 "CHERNOBYL ACCIDENT: ASSESSING THE DATA. ENERGY SERIES NO. 15.
AF: Bent Sørensen.

134/87 "THE D.C. AND THE A.C. ELECTRICAL TRANSPORT IN AsSeTe SYSTEM"
Authors: M.B.El-Den, N.B.Olsen, Ib Høst Pedersen, Petr Visčor

135/87 "INTUITIONISTISK MATEMATIKS METODER OG ERKENDELSESTEORETISKE FORUDSÆTNINGER"
MASTEMATIKSPECIALE: Claus Larsen
Vejledere: Anton Jensen og Stig Andur Pedersen

136/87 "Mystisk og naturlig filosofi: En skitse af kristendommens første og andet møde med græsk filosofi"
Projektrapport af Frank Colding Ludvigsen
Vejledere: Historie: Ib Thiersen
Fysik: Jens Højgaard Jensen

137/87 "HOPMODELLER FOR ELEKTRISK LEDNING I UORDNEDE FASTE STOFFER" - Resume af licentiatafhandling
Af: Jeppe Dyre
Vejledere: Niels Boye Olsen og Peder Voetmann Christiansen.

138/87 "JOSEPHSON EFFECT AND CIRCLE MAP."

Paper presented at The International Workshop on Teaching Nonlinear Phenomena at Universities and Schools, "Chaos in Education". Balaton, Hungary, 26 April-2 May 1987.

By: Peder Voetmann Christiansen

139/87 "Machbarkeit nichtbeherrschbarer Technik durch Fortschritte in der Erkennbarkeit der Natur"

Af: Bernhelm Booss-Bavnbek
Martin Bohle-Carbonell

140/87 "ON THE TOPOLOGY OF SPACES OF HOLOMORPHIC MAPS"

By: Jens Gravesen

141/87 "RADIOMETERS UDVIKLING AF BLODGASAPPARATUR - ET TEKNOLOGIHISTORISK PROJEKT"

Projektrapport af Finn C. Physant

Vejleder: Ib Thiersen

142/87 "The Calderón Projektor for Operators With Splitting Elliptic Symbols"

by: Bernhelm Booss-Bavnbek og
Krzysztof P. Wojciechowski

143/87 "Kursusmateriale til Matematik på NAT-BAS"

af: Mogens Brun Heefelt

144/87 "Context and Non-Locality - A Peircean Approach

Paper presented at the Symposium on the Foundations of Modern Physics The Copenhagen Interpretation 60 Years after the Como Lecture. Joensuu, Finland, 6 - 8 august 1987.

By: Peder Voetmann Christiansen

145/87 "AIMS AND SCOPE OF APPLICATIONS AND MODELLING IN MATHEMATICS CURRICULA"

Manuscript of a plenary lecture delivered at ICMTA 3, Kassel, FRG 8.-11.9.1987

By: Mogens Niss

146/87 "BESTEMMELSE AF BULKRESISTIVITETEN I SILICIUM"

- en ny frekvensbaseret målemetode.

Fysikspeciale af Jan Vedde

Vejledere: Niels Boye Olsen & Petr Viščor

147/87 "Rapport om BIS på NAT-BAS"

redigeret af: Mogens Brun Heefelt

148/87 "Naturvidenskabsundervisning med Samfundsperspektiv"

af: Peter Colding-Jørgensen DLH
Albert Chr. Paulsen

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